View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by National Institute for Fusion Science (NIFS-Repository)

§17. Quasisymmetric Toroidal Plasmas with Large Mean Flows

Sugama, H., Watanabe, T.-H., Nunami, M., Nishimura, S.

Geometric conditions for quasisymmetric toroidal plasmas with large mean flows on the order of the ion thermal speed are investigated [1,2]. Equilibrium momentum balance equations including the inertia term due to the large flow velocity are used to show that, for rotating quasisymmetric plasmas with no local currents crossing flux surfaces, all components $g_{\alpha\beta}$ of the metric tensor should be independent of the toroidal angle in the Boozer coordinates, and consequently these systems need to be rigorously axisymmetric. Unless the local radial currents vanish, the Boozer coordinates do not exist and the toroidal flow velocity cannot take any value other than a very limited class of eigenvalues corresponding to very rapid rotation especially for low beta plasmas.

We consider toroidal plasmas, in which the magnetic field $\mathbf{B}$ is written in terms of the flux coordinates (s, θ, ζ) as $\mathbf{B} = \psi' \nabla s \times \nabla\theta + \chi' \nabla\zeta \times \nabla s = B_s \nabla s + B_\theta \nabla\theta + B_\zeta \nabla\zeta$. Hereafter, we investigate quasisymmetric toroidal systems with large mean flows on the order of the ion thermal velocity v_T. The $\mathcal{O}(v_T)$ equilibrium flow should be tangential to the direction of quasisymmetry, in which the field strength B is uniform. Here, for simplicity, we restrict our consideration to the quasi-axisymmetric case, in which the magnetic field strength B is independent of the toroidal angle ζ,

$$\partial B/\partial\zeta = 0, \qquad B = B(s, \theta), \tag{1}$$

although we can treat general quasisymmetric cases such as quasi-poloidally-symmetric and quasi-helically-symmetric ones in the same way as shown below. When using the perturbative expansion in terms of the drift ordering parameter δ defined by the ratio of the ion thermal gyroradius ρ_T to the equilibrium gradient scale length L, the lowest-order momentum balance equation reduces to $\mathbf{E}_0 + \mathbf{V}_0 \times \mathbf{B}/c = 0$. Here, the lowest-order electric field is given by $\mathbf{E}_0 = -\nabla\Phi_0(s) = -\Phi_0'(s)\nabla s$ and the lowest-order electrostatic potential $\Phi_0(s)$ is a flux-surface function satisfying $e_a\Phi_0/T_a = \mathcal{O}(\delta^{-1})$, where e_a is regarded as a quantity of $\mathcal{O}(\delta^{-1})$. It is found from the lowest-order kinetic equation that the equilibrium flow velocity $\mathbf{V}_0$ of $\mathcal{O}(v_T)$, which consists of the $\mathbf{E} \times \mathbf{B}$ drift and the parallel flow components, should be represented by

$$\mathbf{V}_0 = V^\zeta \frac{\partial \mathbf{x}}{\partial\zeta}, \qquad V^\zeta = -c\frac{\Phi_0'(s)}{\chi'(s)}, \tag{2}$$

and that the following incompressibility condition and other constraints hold: $\nabla\cdot\mathbf{V}_0 = \mathbf{b}\cdot\nabla\mathbf{V}_0\cdot\mathbf{b} = 0$, $\nabla\cdot\partial\mathbf{x}/\partial\zeta = 0$, $\partial\sqrt{g}/\partial\zeta = 0$, $\mathbf{V}_0\cdot\nabla n_a = \mathbf{V}_0\cdot\nabla T_a = 0$, $\mathbf{B}\cdot\nabla T_a = 0$, $n_a = n_a(s,\theta)$, and $T_a = T_a(s)$, where n_a and T_a are the lowest-order density and temperature of the particle species a, respectively. We see that the density n_a and the Jacobian $\sqrt{g} \equiv [\nabla s\cdot(\nabla\theta\times\nabla\zeta)]^{-1}$ are independent of ζ like the field strength B.

The species summation of the equilibrium force balance is written to the lowest order as

$$\left(\sum_a n_a m_a\right)\mathbf{V}_0\cdot\nabla\mathbf{V}_0 = \frac{1}{c}\mathbf{J}\times\mathbf{B} - \nabla p. \tag{3}$$

Then, taking the inner product between Eq. (3) and $\partial\mathbf{x}/\partial\zeta$, we obtain

$$\frac{1}{2}\left(\sum_a n_a m_a\right)(V^\zeta)^2\frac{\partial g_{\zeta\zeta}}{\partial\zeta} = \frac{\chi'}{c}J^s = \frac{B^\theta}{4\pi}\left(\frac{\partial B_\zeta}{\partial\theta} - \frac{\partial B_\theta}{\partial\zeta}\right), \tag{4}$$

In the rigorous axisymmetric case, $\partial g_{\zeta\zeta}/\partial\zeta = 0$ holds although this condition is not trivially satisfied for the quasi-axisymmetric case. If $\partial g_{\zeta\zeta}/\partial\zeta \neq 0$, then Eq. (4) leads to nonzero local radial current $J^s \neq 0$. This gives rise to a serious problem because the quasisymmetric system is considered usually by using the Boozer coordinates while the Boozer coordinates cannot be constructed for the case of $J^s \neq 0$. It is shown in [1] that, for general profiles of the toroidal flow velocity $V^\zeta(s)$, we obtain

$$\partial g_{\zeta\zeta}/\partial\zeta = 0. \tag{5}$$

Then, we find from Eq. (4) that the radial current vanishes, $J^s = 0$, and there exist the Boozer coordinates in which the covariant poloidal and toroidal components, $B_\theta(s)$ and $B_\zeta(s)$, of the magnetic field are flux-surface functions. Here, without loss of generality, we can regard the flux surface (s, θ, ζ) as the Boozer coordinates. In [1], we find from using geometric conditions for toroidal flux surfaces that all metric tensor components $g_{\alpha\beta}$ $(\alpha, \beta = s, \theta, \zeta)$ should be independent of ζ, and that these conditions are satisfied only if the considered system is rigorously axisymmetric.

1) H. Sugama, T.-H. Watanabe, M. Nunami, S. Nishimura, Phys. Plasmas **18**, 082505 (2011).
2) H. Sugama, T.-H. Watanabe, M. Nunami, S. Nishimura, Plasma Phys. Control. Fusion **53**, 024004 (2011).